Klasse 3c – VEKTORGEOMETRIE – MÄRZ 2013 – MD

1) $\vec{n}_E = \vec{v}_1 \times \vec{v}_2 = \begin{pmatrix}4\\0\\-1\end{pmatrix} \times \begin{pmatrix}0\\6\\-2\end{pmatrix} = 2\begin{pmatrix}3\\4\\12\end{pmatrix}$ (1) (1) $E: 3x + 4y + 12z + d = 0$ (1)

7P durch $(0/0/5) \rightarrow 12 \cdot 5 + d = 0 \Rightarrow d = -60$ $\underline{E: 3x + 4y + 12z - 60 = 0}$ (1)

Spurgeraden: $\underline{3x + 4y - 60 = 0}$ (1) $3x + 12z - 60 = 0$ $4y + 12z - 60 = 0$

$\underline{x + 4z - 20 = 0}$ (1) $\underline{y + 3z - 15 = 0}$ (1)

2a) $\vec{n}_E = \vec{v}_g \times \vec{v}_h = \begin{pmatrix}2\\1\\-1\end{pmatrix} \times \begin{pmatrix}1\\-1\\-3\end{pmatrix} = \begin{pmatrix}-4\\5\\-3\end{pmatrix}$ (1) (1) $E: -4x + 5y - 3z + d = 0$

11P durch h: setze A ein: $-4 \cdot 2 + 5 \cdot 1 - 3 \cdot (-6) + d = 0 \Rightarrow d = -15$ (1)

$E: 4x - 5y + 3z + 15 = 0$ (1)

b) $\vec{n}_F = \vec{v}_h \times \vec{n}_E = \begin{pmatrix}1\\-1\\-3\end{pmatrix} \times \begin{pmatrix}-4\\5\\-3\end{pmatrix} = \begin{pmatrix}1\\-1\\-3\end{pmatrix} \times \left[\begin{pmatrix}2\\1\\-1\end{pmatrix} \times \begin{pmatrix}1\\-1\\-3\end{pmatrix}\right] = \begin{pmatrix}18\\15\\1\end{pmatrix}$ (1) (1) $F: 18x + 15y + z + d = 0$

durch h: setze A ein: $18 \cdot 2 + 15 \cdot 1 - 1 \cdot 6 + d = 0 \Rightarrow d = -45$ (1)

$F: 18x + 15y + z - 45 = 0$ (1)

c) $P = g \cap F$, (1)
$$\begin{cases} 18x + 15y + z - 45 = 0 \\ \left.\begin{aligned} x &= 8 + 2t \\ y &= t \\ z &= 1 - t \end{aligned}\right\} \text{in } F \text{ eingesetzt} \end{cases}$$
(1) $\Rightarrow 18(8 + 2t) + 15t + (1 - t) - 45 = 0$

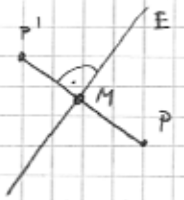
$36t + 15t - t = 45 - 144 - 1$

$t = -2$ (1)

$\vec{OP} = \begin{pmatrix}8\\0\\1\end{pmatrix} - 2 \cdot \begin{pmatrix}2\\1\\-1\end{pmatrix}$ (1) $P = (4/-2/3)$ (1)

d) weil $A \in E \Rightarrow A$ ist Normalprojektion von sich selbst auf E. (1)

4P

3) 

$M = (0+4 / 0+3 / 7-2) \cdot \frac{1}{2} = (2 / \frac{3}{2} / \frac{5}{2})$, $\vec{PP'} = \begin{pmatrix}4\\3\\-9\end{pmatrix}$ (1)

$E: 4x + 3y - 9z + d = 0$ (1)

durch M: $4 \cdot 2 + 3 \cdot \frac{3}{2} - 9 \cdot \frac{5}{2} + d = 0 \Rightarrow d = 10$ (1) $E: 4x + 3y - 9z + 10 = 0$ (1)

16P

4) a)
$$\begin{cases} \vec{p} \cdot \vec{q} = 0 \\ x + 5z = 0 \\ x^2 + y^2 + z^2 = 225 \end{cases}$$
(2)

$\Rightarrow 2x + 2y - z = 0 \Rightarrow 2x + 2y - \frac{x}{5} = 0 \Rightarrow 11x + 10y = 0$

$z = -\frac{x}{5}$ (1) $y = -\frac{11}{10}x$

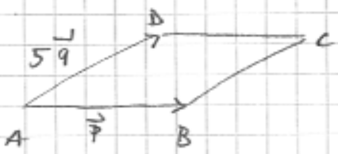
$\Downarrow$

$x^2 + \left(-\frac{11}{10}\right)^2 x^2 + \frac{1}{25}x^2 = x^2\left(1 + \frac{121}{100} + \frac{1}{25}\right) = 225$ (1)

$x^2 = \frac{225}{\frac{9}{4}} \Rightarrow x = \sqrt{\frac{225 \cdot 4}{9}} = 10$ (1) $\vec{p} = \begin{pmatrix}x\\y\\z\end{pmatrix} = 10\begin{pmatrix}1\\-11/10\\-1/5\end{pmatrix} = \begin{pmatrix}10\\-11\\-2\end{pmatrix}$ (1)

b) $|\vec{AB}| = |\vec{AD}| \Rightarrow |\vec{p}| = |k||\vec{q}|$ (1) $\quad |\vec{p}| = \sqrt{10^2+11^2+5^2} = 15 = k \cdot \sqrt{2^2+2^2+1} = 3k = |k||\vec{q}|$

$\Rightarrow k = \frac{15}{3} = 5$ (1)



$\vec{OD} = \vec{OA} + 5\vec{q} = \begin{pmatrix}-2\\1\\7\end{pmatrix} + 5\begin{pmatrix}2\\2\\-1\end{pmatrix} = \begin{pmatrix}8\\11\\2\end{pmatrix} \quad D = (8/11/2)$ (1)

$\vec{OB} = \vec{OA} + \vec{p} = \begin{pmatrix}-2\\1\\7\end{pmatrix} + \begin{pmatrix}10\\-11\\-2\end{pmatrix} = \begin{pmatrix}8\\-10\\5\end{pmatrix} \quad B = (8/-10/5)$ (1)

$\vec{OC} = \vec{OA} + \vec{p} + 5\vec{q} = \begin{pmatrix}-2\\1\\7\end{pmatrix} + \begin{pmatrix}10\\-11\\-2\end{pmatrix} + 5\begin{pmatrix}2\\2\\-1\end{pmatrix} = \begin{pmatrix}18\\0\\0\end{pmatrix} \quad C = (18/0/0)$ (1)

c) $V = \frac{1}{3} G h \qquad G = |\vec{p} \times 5\vec{q}| = 5 \left| \begin{pmatrix}10\\-11\\-2\end{pmatrix} \times \begin{pmatrix}2\\2\\-1\end{pmatrix} \right| = 5 \left| \begin{pmatrix}15\\6\\42\end{pmatrix} \right| = 5 \cdot 45 = 225$ (2)

h: Abstand ABCD zu $O(0/0/0) \Rightarrow$ HNF, d ablesen! $\quad (\vec{n_E} = \vec{p} \times \vec{q})$

E: $15x + 6y + 42z + d = 0$ durch $A(-2/1/7)$ mit $|\vec{p} \times \vec{q}| = 45$

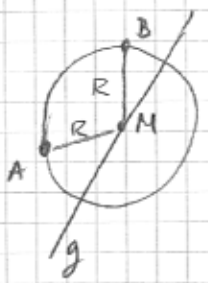
$-15 \cdot 2 + 6 \cdot 1 + 42 \cdot 7 + d = 0 \Rightarrow d = -270$

E: $15x + 6y + 42z - 270 = 0$ normiert $\Longrightarrow \frac{15}{45}x + \frac{6}{45}y + \frac{42}{45}z - \frac{270}{45} = 0$

E: $\frac{1}{3}x + \frac{2}{15}y + \frac{14}{15}z - 6 = 0 \Rightarrow e = h = |d| = 6$ (Abstand ABCD – $O(0/0/0)$)
(1)

$V = \frac{1}{3} G \cdot h = \frac{1}{3} |\vec{p} \times 5\vec{q}| \cdot e = \frac{1}{3} \cdot 225 \cdot 6 = 450$ (1)
(1)

5) a)



$|\vec{AM}| = |\vec{BM}| \Rightarrow \left| \begin{pmatrix}8+8t+6\\-2-4t-5\\1+t-6\end{pmatrix} \right|$ (1) $= \left| \begin{pmatrix}8+8t-4\\-2-4t+5\\1+t-4\end{pmatrix} \right| \Rightarrow t = -1$ (1)

(8P)

$\vec{AM} = \begin{pmatrix}8-8+6\\-2+4-5\\1-1-6\end{pmatrix} = \begin{pmatrix}6\\-3\\-6\end{pmatrix} \qquad \vec{OM} = \vec{OA} + \vec{AM} = \begin{pmatrix}-6\\5\\6\end{pmatrix} + \begin{pmatrix}6\\-3\\-6\end{pmatrix} = \begin{pmatrix}0\\2\\0\end{pmatrix}$ (1)

$r = |\vec{AM}| = \sqrt{36+25+36} = 9$ (1) $\qquad \vec{BM} = \begin{pmatrix}8-8-4\\-2+4+5\\1-1-4\end{pmatrix} = \begin{pmatrix}-4\\7\\-4\end{pmatrix}$

b) $\alpha = \arccos \frac{|\vec{AM} \cdot \vec{BM}|}{|\vec{AM}||\vec{BM}|}$ (1) $= \arccos \frac{\left|\begin{pmatrix}6\\-3\\-6\end{pmatrix}\begin{pmatrix}-4\\7\\-4\end{pmatrix}\right|}{81}$ (1) $= \arccos \frac{|-24-21+24|}{81} =$

$= \arccos \frac{21}{81} = \arccos \frac{7}{27} = 75^0$ (1)

weil $\widehat{E,F} = \widehat{\vec{n_E}, \vec{n_F}} = \widehat{\vec{AM}, \vec{BM}}$ (1)