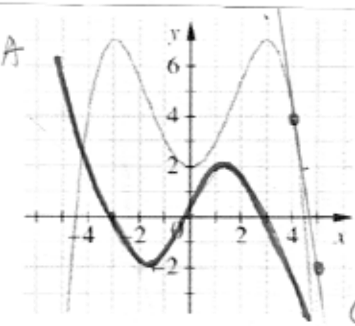
Mathe – Prüfung – 7/11/12 – 3c – MD

1)

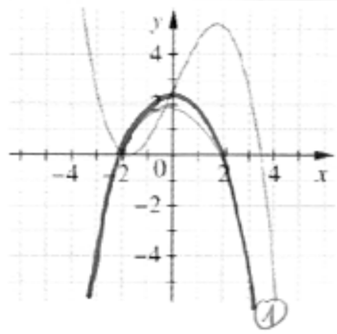
6

A



1

B



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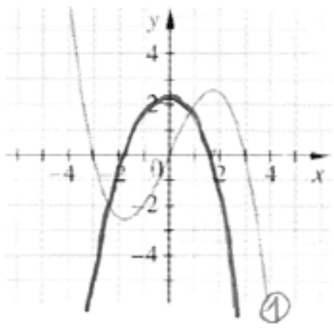
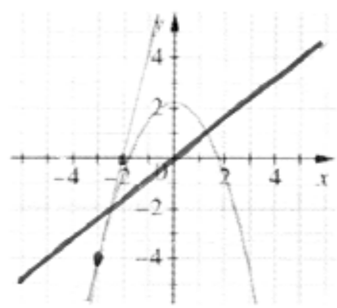
C



1

D



$A' = C$ 1

$B' = C' = D$ 2

(siehe Diagramme mit Ableitung)

2) 4

$f_A'(4) \cong \frac{-2-4}{5-4} = -6$ 2 $\qquad f_D'(-3) = \frac{0-(-4)}{-2-(-3)} = 4$ 2

3) 18

a) $f'(x) = \left((3-x)x - 5x^3\right)' = -x + (3-x) - 15x^2 = -15x^2 - 2x + 3$ 1

b) $f'(t) = (\ln 4x + e^x)' = 0 \quad \left[f'(x) = \frac{1}{x} + e^x\right]$ 1

c) $f'(x) = \left(\frac{x^2}{\sin x}\right)' \overset{2}{=} \frac{2x\sin x - x^2\cos x}{\sin^2 x} \overset{1}{=} \frac{x}{\sin x}\left(2 - \frac{x}{\tan x}\right)$

d) $f'(k) = \left(\sqrt{k}(k^2-1)\right)' \overset{2}{=} \frac{1}{2\sqrt{k}}(k^2-1) + \sqrt{k}\cdot 2k = \frac{1}{2\sqrt{k}}\left(k^2 - 1 + 2\cdot 2k^2\right) \overset{1}{=} \frac{1}{2\sqrt{k}}\left(5k^2-1\right)$

e) $f'(p) = \left(p^3 \ln 3p\right)' \overset{1}{=} 3p^2 \ln 3p + p^3 \cdot \frac{1}{3p}\cdot 3 = 3p^2 \ln 3p + p^2 \overset{1}{=} p^2(3\ln 3p + 1)$

f) $f'(u) = \left(\frac{2+e^u}{2-e^u}\right)' \overset{2}{=} \frac{e^u(2-e^u) - (2+e^u)(-e^u)}{(2-e^u)^2} = \frac{4e^u}{(2-e^u)^2}$ 1

i) $f'(t) = \left(\sin\sqrt{1/t}\right)' \overset{2}{=} \cos\sqrt{\frac{1}{t}} \cdot \frac{1}{2\sqrt{1/t}} \cdot (-1)t^{-2} = \frac{-\sqrt{t}}{2t^2}\cos\sqrt{\tfrac{1}{t}} = -\frac{1}{2}t^{-3/2}\cos\sqrt{\tfrac{1}{t}}$ 1

j) $f'(x) = \left(\ln(\ln x)\right)' \overset{1}{=} \frac{1}{\ln x}\cdot\frac{1}{x} = (x\ln x)^{-1}$

k) $f'(t) = 0$ 1 $\left[f'(x) = e^{\sin x\cos x}\cdot\left(\underbrace{\cos x\cos x - \sin x \sin x}_{1-2\cos^2 x}\right) = (1-2\cos^2 x)\,e^{\sin x\cos x}\right.$

4) 8

a) $f'(x) = \frac{1}{2\sqrt{(x-1)(x+2)}} \overset{1}{\cdot} \left[1(x+2) + (x-1)\cdot 1\right] = \frac{2x+1}{2\sqrt{(x-1)(x+2)}}$ 1

$f'(2) = \frac{2\cdot 2+1}{2\sqrt{(2-1)(2+2)}} \overset{1}{=} \frac{5}{2\sqrt{4}} = \frac{5}{4}$ 1

b) $f'(x) = \frac{1 \cdot e^{ax} - x \cdot a e^{ax}}{e^{2ax}} = \frac{1-ax}{e^{ax}}$ 1

$f'(\frac{1}{a}) \overset{1}{=} \frac{1 - a \cdot 1/a}{e^{a \cdot 1/a}} = \frac{0}{e^1} = \underline{\underline{0}}$ 1

5) $\lim_{h \to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \to 0} \frac{\frac{(x+h)^2+1}{x+h} - \frac{x^2+1}{x}}{h}$ 1 $=$

6

$= \lim_{h \to 0} \frac{(x+h)^2 \cdot x + x - (x^2+1)(x+h)}{h(x+h) \cdot x} \overset{1}{=} \lim_{h \to 0} \frac{\cancel{x^3} + xh^2 + 2x^2h + \cancel{x} - \cancel{x^3} - x^2h - \cancel{x} - h}{xh(x+h)} \overset{1}{=}$

$\overset{1}{=} \lim_{h \to 0} \frac{xh^2 + x^2h - h}{hx(x+h)} \overset{1}{=} \lim_{h \to 0} \frac{\cancel{h}}{\cancel{(x+h)}} + \frac{x}{x+h} - \frac{1}{x(h+x)} \underset{h=0}{\overset{1}{=}} 1 - \frac{1}{x^2}$

6)

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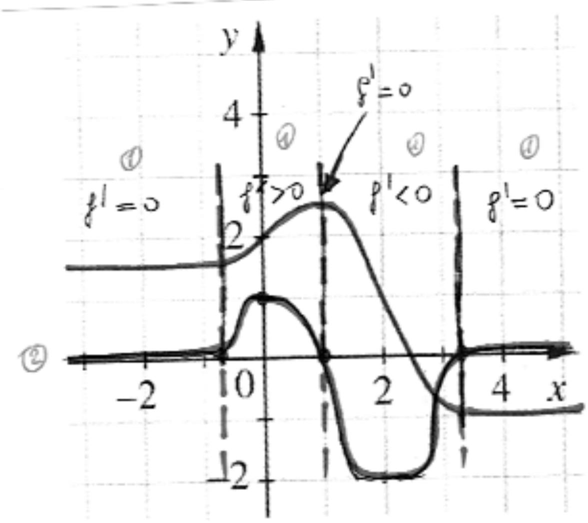

7) Stetigkeit: $y=2x$ und $y=x^2-1$ haben $\mathbb{D}=\mathbb{R}$. Einziger Ort
1
zu testen ist $x=1$.

7

$f(1) \overset{1}{=} 2 \cdot 1 = 2$ $\quad \lim_{x \to 1^-} 2x \overset{1}{=} 2$ $\quad \lim_{x \to 1^+} x^2 - 1 \overset{1}{=} 0$ $\Rightarrow$ nicht stetig in $x=1$.

Differenzierbarkeit: $y=2x$ und $y=x^2-1$ sind überall diffbar.
Einziger Ort zu testen ist $x=1$. Weil $f(x)$ in $x=1$
1
nicht stetig $\Rightarrow$ f in $x=1$ auch nicht diffbar.
Es existieren aber die linke und die rechte Ableitungen:

$\lim_{h \to 0^-} \frac{2(x+h) - 2x}{h} = \lim_{h \to 0^-} \frac{\cancel{2x} + 2h - \cancel{2x}}{h} = 2$ linke Ableitung 1

$\lim_{h \to 0^+} \frac{(x+h)^2 - 1 - x^2 + 1}{h} = \lim_{h \to 0^+} \frac{\cancel{x^2} + 2xh + h^2 \cancel{-1} \cancel{-x^2} \cancel{+1}}{h} = 2x$ rechte Ableitung. 1