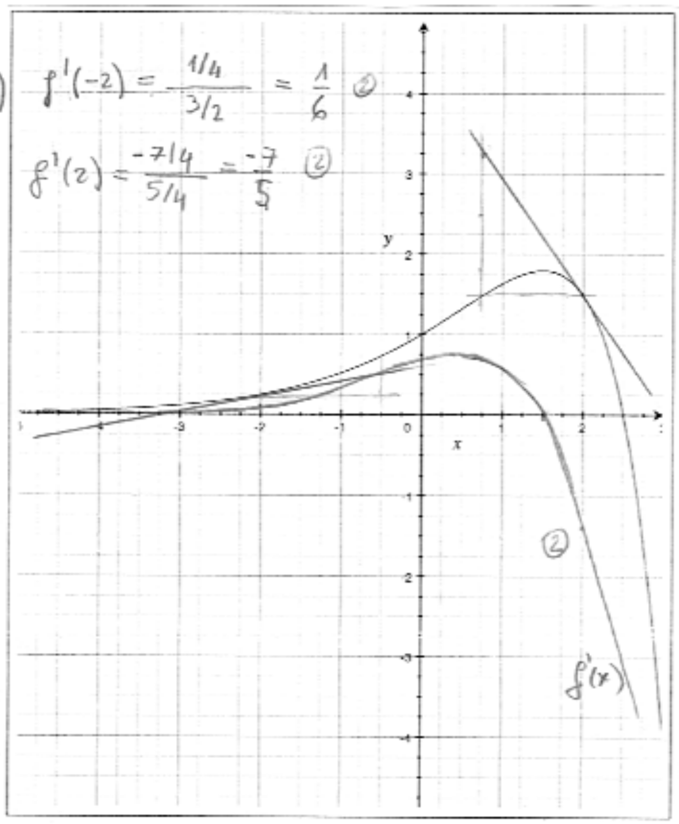
M-Prüfung – 3C – Dez. 2012 – MD

TEIL A

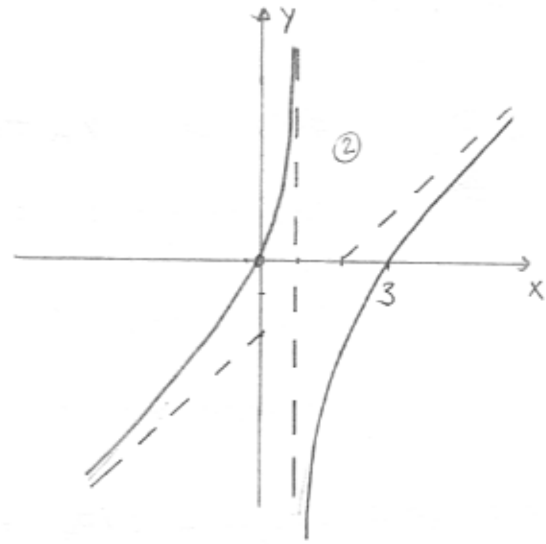
(6)

1) $f'(-2) = \frac{1/4}{3/2} = \frac{1}{6}$ (2)

$f'(2) = \frac{-7/4}{5/4} = \frac{-7}{5}$ (2)



2) Kurvendiskussion



2) a) (1) $\mathbb{D} = \mathbb{R} \setminus \{1\}$ mit $f(x) = x - \frac{2x}{x-1}$

(11) b) (1) $f(x) = 0 = x - \frac{2x}{x-1} \iff x^2 - x = 2x$, $x^2 = 3x$; $x = 0$, $x = 3$ NS

c) (1) $f(0) = 0$

d) (1) $\lim\limits_{x \to \infty} x - \frac{2x}{x-1} = \lim\limits_{x \to +\infty} x - \frac{x \cdot 2}{x(1-\frac{1}{x})} = \infty$

(1) $\lim\limits_{x \to -\infty} x - \frac{2x}{x-1} = -\infty$

(1) $\lim\limits_{x \to 1^+} x - \frac{2x}{x-1} = \lim\limits_{x \to 1^+} 1 - \frac{2}{x-1} = -\infty$

(1) $\lim\limits_{x \to 1^-} x - \frac{2x}{x-1} = \lim\limits_{x \to 1^-} 1 - \frac{2}{x-1} = +\infty$

e) $f'(x) = 0$

$f'(x) = 1 - \frac{2(x-1) - 2x \cdot 1}{(x-1)^2} = \frac{(x-1)^2 - 2(x-1) + 2x}{(x-1)^2} =$

(2) $= \frac{x^2 - 2x + 1 - \cancel{2x} + 2 + \cancel{2x}}{(x-1)^2} = \frac{x^2 - 2x + 3}{(x-1)^2} = 0$

$\Rightarrow x^2 - 2x + 3 = 0 \Rightarrow x = \frac{2}{2} \pm \frac{1}{2}\sqrt{\underbrace{4 - 4 \cdot 2}_{<0}} \Rightarrow$ nie!

f) Asymptoten $\quad$ für $x \to \pm\infty \quad f(x) \longrightarrow x + q$

(2) $\dfrac{(x-1)\cdot x - 2x}{x-1} = \dfrac{x^2-3x}{x-1} \qquad x^2 - 3x : x-1 = x - 2 + \dfrac{2}{x-1}$

$$\begin{array}{l} x^2 - 3x \\ \underline{-(x^2 - x)} \\ \quad -2x \\ \underline{-(-2x+2)} \\ \quad -2 \end{array}$$

(*) (VORNE) $\qquad \Rightarrow q = -2$

3)  Zielgrösse: $K = \pi R^2 \cdot p_M + 2\pi R h\, p_P$ (2)

Nebenbedingung: $p_M \overset{(1)}{=} 4p_P$; $V = \pi R^2 h \Rightarrow h \overset{(1)}{=} \dfrac{V}{\pi R^2}$, $V = 10^{-3} m^3$

Zielfunktion: $K(R) = \pi R^2 4 \cdot p_P + \dfrac{2\pi R V}{\pi R^2} \cdot p_P = 4\pi p_P R^2 + \dfrac{2Vp_P}{R}$ (1)

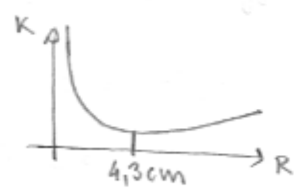
Def-Bereich: $h \in [0; \infty[$; $R \in [0; \infty[$ (1)

$K'(R) = 0 \overset{(1)}{=} 8\pi p_P \cdot R - \dfrac{2Vp_P}{R^2} \Rightarrow 8\pi p_P R = \dfrac{2Vp_P}{R^2} \Rightarrow R = \sqrt[3]{\dfrac{2V\cancel{p_P}}{8\pi \cancel{p_P}}} = \sqrt[3]{\dfrac{V}{4\pi}}$ (1)

$h = \dfrac{V}{\pi R^2} = \dfrac{V}{\pi} \cdot \left(\dfrac{4\pi}{V}\right)^{2/3} = \left(\dfrac{V^{3/2} \cdot 4\pi}{\pi^{3/2} \cdot V}\right)^{2/3} = \left(V^{1/2}\, 16^{1/2} \cdot \pi^{-1/2}\right)^{2/3} = \sqrt[3]{\dfrac{16V}{\pi}}$ (1) (1)

Sicher ein Min weil $K(R) \to \infty$ für $R \to 0$ und $K(R) \to \infty$ für $R \to \infty$!

Einsetzen ($V = 1000\ cm^3$) $\qquad R = 4{,}3 cm \quad$ (1) $\quad h = 17{,}2 cm$



4)a) $(e^{\sin t})' \underset{(2)}{=} \cos t \cdot e^{\sin t}$ $\qquad$ b) $(\arcsin(k^5))' \underset{(2)}{=} \dfrac{5k^4}{\sqrt{1-k^{10}}}$

c) $\left(\ln s \cdot \sqrt{s^2+1}\right)' = \dfrac{\sqrt{s^2+1}}{s} + \ln s \cdot \dfrac{\cancel{2}s}{\cancel{2}\sqrt{s^2+1}} = \sqrt{1+\frac{1}{s^2}}\left(1 + \dfrac{\ln s}{1+\frac{1}{s^2}}\right)$ (3)

d) $\left[\ln\left(\arctan\sqrt{x}\right)\right]' = \dfrac{1}{\arctan\sqrt{x}} \cdot \dfrac{1}{1+x} \cdot \dfrac{1}{2\sqrt{x}} = \dfrac{1}{2\sqrt{x}(1+x)\arctan\sqrt{x}}$ (3)

5)a) $f_t'(x) = e^{x^2} + (x-t)\cdot e^{x^2} \cdot 2x \overset{(1)}{=} e^{x^2}\left(1 + 2x(x-t)\right) \overset{(1)}{=} 0 \qquad (e^{x^2} \text{ nie } = 0!)$

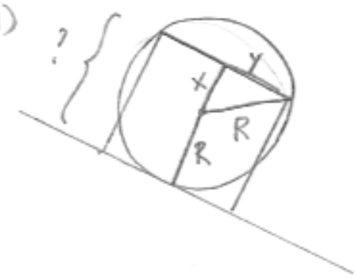
$\Leftrightarrow 1 + 2x^2 - 2xt = 0 \Leftrightarrow x^2 - xt + \frac{1}{2} = 0$ (1)

$x = \left(t \pm \sqrt{t^2 - 4 \cdot \frac{1}{2}}\right)\frac{1}{2} = \frac{1}{2}\left(t \pm \sqrt{t^2-2}\right)$ (1)

b) $x \in \emptyset \Leftrightarrow t^2 - 2 < 0 \Leftrightarrow t^2 < 2 \Leftrightarrow t \in [-\sqrt{2}; \sqrt{2}]$ (1) (2)

TEIL B

(10) 1)



① Zielgrösse : $A = 2y \cdot (R+x)$

① Nebenbedingung : $x^2+y^2=R^2 \Rightarrow y=\sqrt{R^2-x^2}$

② Zielfunktion ; $A(x) = 2(R+x)\sqrt{R^2-x^2}$

① Def.-Bereich ; $x \in [0 ; R]$

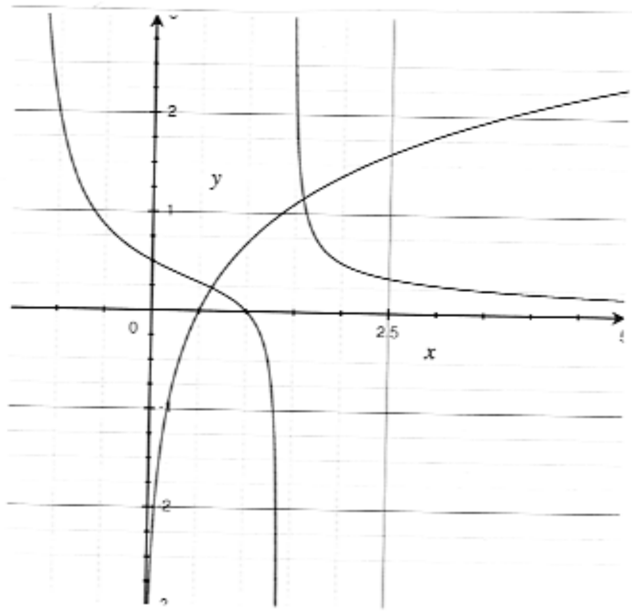
$\text{solve}\left(d\left(2(R+x)\sqrt{R^2-x^2}, x\right)=0, x\right) \Rightarrow x = R/2$ ③ ①

① Abstand $d = x+R = \frac{R}{2}+R = \underline{\underline{\frac{3}{2}R}}$ Fläche maximal wenn $d=\frac{3}{2}R$

(6) 2) a) Schnittspunkte : $f(x) = g(x)$!

exp ▸ list $\left(\text{solve}\left(\ln(2x) = \frac{x-1}{x^2-2}, x\right), x\right) \rightarrow t$

⇒ Lösungen werden gespeichert als $t = \begin{pmatrix} t_1 \\ t_2 \end{pmatrix}$
mit $t[1] = 0{,}63$ und $t[2] = 1{,}58$ ②



b) $\ln(2x)$ monoton steigend
d.h Ableitung immer > 0.
$\frac{x-1}{x^2-2}$ monoton fallend, d.h
Ableitung immer < 0.

Schnittwinkel :

$\alpha_1 = \text{abs}\left(\arctan\left(d(\ln 2x, x)\right)\middle| x=t[1] - \arctan\left(d\left(\frac{x-1}{x^2-2}\right), x\right)\middle| x=t[1]\right)$ ②

$\alpha_1 = 82°$ (auch 98°)

$\alpha_2 = \text{abs}\left(\arctan\left(d(\ln 2x, x)\right)\middle| x=t[2] - \arctan\left(d\left(\frac{x-1}{x^2-2}\right), x\right)\middle| x=t[2]\right)$ ②

$\alpha_2 = 111°$ (auch 69°)