VEKTORGEOMETRIE – 3C – 01/2013 – MD

1) $\vec{a}\cdot\vec{b} = \begin{pmatrix}1\\2\\0\end{pmatrix}\cdot\begin{pmatrix}-3\\4\\-5\end{pmatrix} = -3+8+0 = 5$ ②

$\vec{a}\times\vec{b} = \begin{pmatrix}1\\2\\0\end{pmatrix}\times\begin{pmatrix}-3\\4\\-5\end{pmatrix} = \begin{pmatrix}10\\-5\\-10\end{pmatrix}$ $\quad\vec{b}\times\vec{a} = \begin{pmatrix}-3\\4\\-5\end{pmatrix}\times\begin{pmatrix}1\\2\\0\end{pmatrix} = \begin{pmatrix}10\\-5\\-10\end{pmatrix}$ ②

$p = \frac{|\vec{b}\cdot\vec{a}|}{|\vec{a}|} = \frac{5}{\sqrt{5}} = \sqrt{5}$ ②

2) a) $\begin{pmatrix}6\\0\\-1\end{pmatrix} = k\begin{pmatrix}-4\\y\\z\end{pmatrix} \Rightarrow k = \frac{6}{-4} = -\frac{3}{2}$ ① $\Rightarrow y = 0$ (1/2)

$z = -\frac{1}{k} = \frac{2}{3}$ (1/2)

$\vec{b} = \begin{pmatrix}-4\\0\\2/3\end{pmatrix}$

b) $\begin{pmatrix}4\\y\\0\end{pmatrix} = k\begin{pmatrix}x\\-9\\1\end{pmatrix} \Rightarrow k = \frac{0}{1} = 0$ (1/2) $\Rightarrow y = 0$ (1/2) $x \in \emptyset$ unmöglich ①

3) a) $\vec{a} = \begin{pmatrix}2\\-1\\2\end{pmatrix}$ $\vec{b} = \begin{pmatrix}x\\1\\2\end{pmatrix}$ $\quad\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cdot\cos\alpha$ ①

$|\vec{a}| = 3$ $\quad|\vec{b}| = \sqrt{x^2+5}$ $\quad 2x+3 = 3\cdot\sqrt{x^2+5}\cdot 1/2$ ①

$\cos 60 = \frac{1}{2}$ $\quad 4x+6 = 3\sqrt{x^2+5}$

$16x^2+36+48x = 9x^2+45$

$\vec{a}\cdot\vec{b} = 2x-1+4$ $\quad 7x^2+48x-9 = 0$ ①

$x = \frac{-48 \pm \sqrt{48^2-4\cdot 7\cdot 9}}{14} = \frac{-48 \pm \sqrt{2052}}{14}$ ①

b) $\vec{e}_2 = \begin{pmatrix}0\\0\\1\end{pmatrix}$ $\quad\vec{a} = \begin{pmatrix}2\\-1\\2\end{pmatrix}$ $\quad\alpha = \arccos\frac{2\cdot 1}{\sqrt{9}} = \arccos\frac{2}{3}$ ①

4) a) $\det\begin{pmatrix}k-2 & 1 & k\\1 & -1 & -1\\4 & 4 & 4\end{pmatrix} = 0$ ①

$-(k-2)\cdot 4 + 4k - k + k^2 + (k-2)\cdot 4 - 4 = 0$ ①

$k^2+3k-4 = 0$

$(k+4)(k-1) = 0 \Rightarrow k \in \{-4; 1\}$ ②

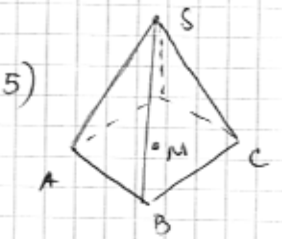
b) $k=1$ $\quad\vec{a} = \begin{pmatrix}-1\\1\\1\end{pmatrix}$ $\quad\vec{b} = \vec{c} = \begin{pmatrix}1\\-1\\4\end{pmatrix}$ ①

2 Vektoren sind immer komplanar. Da $\vec{a}$ und $\vec{b} = \vec{c}$ nicht parallel gibt es <u>keine</u> lineare Kombination.

$K = -4 \Rightarrow \quad \vec{a} = \begin{pmatrix} -6 \\ 1 \\ -4 \end{pmatrix} \quad \vec{b} = \begin{pmatrix} 1 \\ -1 \\ 4 \end{pmatrix} \quad \vec{c} = \begin{pmatrix} -4 \\ -1 \\ 4 \end{pmatrix}$ ①

① $\begin{cases} -6 = x \cdot 1 - 4 \cdot y \\ 1 = -x - y \\ -4 = 4x + 4y \end{cases} \Rightarrow \quad x = -2 \quad y = 1$ ①

$\underline{\vec{a} = -2\vec{b} + \vec{c}}$ ①

5) 

$\vec{OM} = \vec{OA} + \frac{1}{2}\vec{AC} = \begin{pmatrix} 8 \\ -2 \\ 3 \end{pmatrix} + \frac{1}{2}\begin{pmatrix} -6-8 \\ 6+2 \\ -4-3 \end{pmatrix} = \begin{pmatrix} 1 \\ 2 \\ -1/2 \end{pmatrix}$ ②

$M = (1/2/-1/2)$

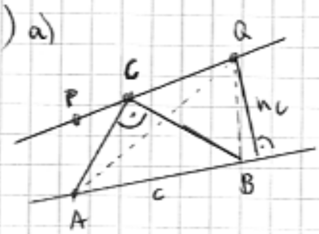
$K\,|\vec{AB} \times \vec{BC}| = 10$ ①

$K = \dfrac{10}{\left|\begin{pmatrix} 0-8 \\ 4+2 \\ -8-3 \end{pmatrix} \times \begin{pmatrix} -6-0 \\ 6-4 \\ -4+8 \end{pmatrix}\right|} = \dfrac{10}{\left|2\begin{pmatrix} -8 \\ 6 \\ -11 \end{pmatrix} \times \begin{pmatrix} -3 \\ 1 \\ 2 \end{pmatrix}\right|} = \dfrac{10}{\left|2\begin{pmatrix} 23 \\ 49 \\ 10 \end{pmatrix}\right|} =$ ②

$= \dfrac{5}{\sqrt{3030}}$ ①

$\vec{OS} = \vec{OM} + K \cdot \vec{AB} \times \vec{AC} = \begin{pmatrix} 1 \\ 2 \\ -1/2 \end{pmatrix} + \dfrac{5 \cdot 2}{\sqrt{3030}}\begin{pmatrix} 23 \\ 49 \\ 10 \end{pmatrix}$ ②

6) $g: \vec{r} = \vec{OA} + t\,\vec{AB} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 5-2 \\ 0+1 \\ 0-5 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 5 \end{pmatrix} + t\begin{pmatrix} 3 \\ 1 \\ -5 \end{pmatrix}$ ③

7) a) 

$\vec{AQ} = \begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} \quad \vec{AB} = \begin{pmatrix} 5 \\ 12 \\ 2 \end{pmatrix}$

$\vec{OC} = \begin{pmatrix} -1 \\ 1 \\ 4 \end{pmatrix} + t\begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -1 \\ 1 \\ 4+t \end{pmatrix}$ ①

$\vec{CA} \cdot \vec{CB} = 0$ ① $\quad \vec{CA} = \begin{pmatrix} 2 \\ -3 \\ 4 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 4+t \end{pmatrix} = \begin{pmatrix} 3 \\ -4 \\ -t \end{pmatrix}$

$\vec{CB} = \begin{pmatrix} 7 \\ 9 \\ 6 \end{pmatrix} - \begin{pmatrix} -1 \\ 1 \\ 4+t \end{pmatrix} = \begin{pmatrix} 8 \\ 8 \\ 2-t \end{pmatrix}$

$\Rightarrow 24 - 32 - t(2-t) = -8 - 2t + t^2 = t^2 - 2t - 8 \overset{①}{=} (t-4)(t+2) = 0 \quad \begin{cases} t = 4 & C_1 = (-1/1/8) \\ ② & \\ t = -2 & C_2 = (-1/1/2) \end{cases}$

b) $h_c = \dfrac{|\vec{AQ} \times \vec{AB}|}{|\vec{AB}|}$ ① $= \dfrac{\left|\begin{pmatrix} -3 \\ 4 \\ 1 \end{pmatrix} \times \begin{pmatrix} 5 \\ 12 \\ 2 \end{pmatrix}\right|}{\left|\begin{pmatrix} 5 \\ 12 \\ 2 \end{pmatrix}\right|}$ ① $= \dfrac{\left|\begin{pmatrix} 4 \\ -11 \\ -56 \end{pmatrix}\right|}{\sqrt{25+144+4}} = \dfrac{\sqrt{3273}}{\sqrt{173}}$ ①

8) $B = (6/6/0)$

$C = (0/0/6)$ ① $\quad R: \vec{r} = \begin{pmatrix} 0 \\ 0 \\ 6 \end{pmatrix} + t\begin{pmatrix} 6 \\ 6 \\ -6 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix} + t\begin{pmatrix} 3 \\ 3 \\ 6 \end{pmatrix}$ ③

$A = (0/0/0)$ $\quad SP = (2/2/4)$ ①

$S = (3/3/6)$ ① $\begin{cases} 6t_1 = 3t_2 \\ -6t_1 + 6 = 6t_2 \end{cases} \quad t_1 = 1/3 \quad t_2 = 2/3$ ②