Integralrechnung – MD – 4c

2p

1) a) $\int 17\,dx = 17x + K$ 1 b) $\int at\,dt = \frac{1}{2}at^2 + K$ 1 c) $\int (3z^2 - 4z + \frac{1}{z})\,dz = z^3 - 2z^2 + \ln|z| + K$ 3

d) $\int \left(\frac{3}{x} - \frac{2}{x^2} + \frac{3}{x^3}\right) dx = 3\ln|x| + 2x^{-1} - \frac{3}{2}x^{-2} + K$ 3

e) $\int \left(\frac{a}{x} + \frac{b}{x^2}\right) dx = a\ln|x| - bx^{-1} + K$ 2 f) $\int (t^{1/2} - t^{-1/2})\,dt = \frac{2}{3}t^{3/2} - 2t^{1/2} + K$ 2

g) $y^2 - 1 : y^2 + 1 = 1$
$r = -2$
$\int 1\,dy - \int \frac{2}{1+y^2}\,dy = y - 2\arctan y + K$ 3

h) $\int \frac{dk}{k^2+k+3} = \int \frac{dk}{(k+\frac{1}{2})^2 + \frac{11}{4}} = \int \frac{4/11\,dk}{(\frac{2k}{\sqrt{11}} + \frac{1}{\sqrt{11}})^2 + 1} = \frac{4}{11} \cdot \frac{2}{\sqrt{11}} \int \frac{\frac{\sqrt{11}}{2}\,dk}{(\frac{2}{\sqrt{11}}k + \frac{1}{\sqrt{11}})^2 + 1} =$

$= \frac{4}{11} \cdot \frac{\sqrt{11}}{2} \arctan\left(\frac{2}{\sqrt{11}}k + \frac{1}{\sqrt{11}}\right) + K$ 3

i) $\int \frac{e^m}{1+e^{2m}}\,dm = \int \frac{e^m}{1+(e^m)^2}\,dm = \arctan(e^m) + K$ 3

2) $\int_1^x \frac{1}{t}\,dt = \ln|x| - \ln 1 = \ln|x| = \ln\left|\frac{ax}{a}\right| = \ln|ax| - \ln|a| = \int_a^{ax} \frac{1}{t}\,dt$

3p 1 1 1

3) $I = 4\int_0^9 \sqrt{9-x}\,dx = -4 \cdot \frac{2}{3}\int_0^9 -\frac{3}{2}(9-x)^{1/2}\,dx = -\frac{8}{3}(9-x)^{3/2}\Big|_0^9 = -\frac{8}{3}\left(0 - 9^{3/2}\right) =$ 1 1

8p $= \frac{8}{3} \cdot (3^2)^{3/2} = \frac{8}{3} \cdot 3^3 = 72$ 1

Diagramm 3p

4) $f(x) = \ln(1+x)$ $f(0) = \ln(1) = 0$ $f'''(x) = \frac{2}{(1+\cancel{x})^3}$ $f'''(0) = 2$

6p $f'(x) = \frac{1}{1+x}$ $f'(0) = 1$ $f''''(x) = \frac{-6}{(1+x)^4}$ $f''''(0) = -6$

$f''(x) = \frac{-1}{(1+x)^2}$ $f''(0) = -1$

3

$f(x) = x - \frac{x^2}{2} + \frac{2x^3}{3!} \pm \frac{6x^4}{4!} = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} \ldots = \sum_{n=1}^{\infty} \frac{(-1)^{n+1} x^n}{n}$

$f(2) = f(1+1) = 1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} \overset{1}{=} \frac{12-6+4-3}{12} = \frac{7}{12}$ 1

$f(2) = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}$ 1

5) $\overline{F} = \frac{1}{4}\int_0^4 F(t)\,dt = \frac{K}{4}\int_0^4 (2t+1)^{-1/2}\,dt \overset{1}{=} \frac{K}{4}\frac{1}{2}\cdot 2\int_0^4 2\cdot\frac{1}{2}(2t+1)^{-1/2}\,dt =$

(5p) $= \frac{K}{4}\,(2t+1)^{1/2}\Big|_0^4 = \frac{K}{4}\left(9^{1/2} - 1^{1/2}\right) = \frac{K}{4}\cdot 2 = \frac{K}{2}$ 1 1

$F(t) = \frac{K}{2} \Rightarrow \frac{K}{\sqrt{2t+1}} = \frac{K}{2}$ 1 $\Rightarrow \sqrt{2t+1} = 2 \Rightarrow t = \frac{3}{2}$ 1

6) a) $f'(x) = x^2 - x - 2$

$f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x + K$ 3

(8p)

$f(1) = \frac{1}{3} - \frac{1}{2} - 2 + K = 8$

$K = 8 + 2 + \frac{1}{2} - \frac{1}{3} = 10 + \frac{1}{2} - \frac{1}{3} = \frac{61}{6}$ 1

$f(x) = \frac{x^3}{3} - \frac{x^2}{2} - 2x + \frac{61}{6}$

b) $g'(t) = 2e^{-t}$

$g(t) = -2e^{-t} + K$ 2

$g(t) = -2e^{-t} - 2 + 4e^{-2}$

$g(0) = -2 + K = g(2)\cdot 2 = -2e^{-2}\cdot 2 + K\cdot 2$

$K = -2 + 4e^{-2}$ 2

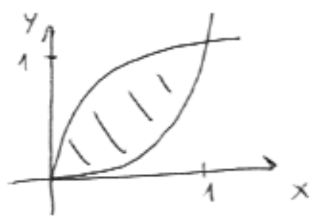
7) $f'(x) = x^3 - x \overset{1}{=} 0 \Rightarrow \begin{array}{l} x = 0 \\ x = \pm 1 \end{array}$ 2

$f''(x) = 3x^2 - 1$ 1

(6p) $f''(0) = -1 < 0 \Rightarrow$ Max 1

$f''(\pm 1) = 2 > 0 \Rightarrow$ Min! 1

8)



(5p)

$I = \int_0^1 (x^{1/n} - x^n)\,dx$ 1 $= \left(\frac{n}{n+1}\,x^{\frac{n+1}{n}} - \frac{1}{n+1}\,x^{n+1}\right)\Big|_0^1 =$ 2

$= \frac{n}{n+1} - \frac{1}{n+1} = \frac{n-1}{n+1}$ 1

$\lim_{n\to\infty} I = \lim_{n\to\infty} \frac{n-1}{n+1} = 1$ // 1