Musterlösung Prüfung Integralrechnung 2, Klasse 4c, November 2013

1a) $$\int x\sin(x^2)\,dx = \frac{1}{2}\int \sin x^2 \cdot \underbrace{2x\,dx}_{du} \overset{1}{=} \frac{1}{2}\int \sin u\,du \overset{1}{=} -\frac{1}{2}\cos x^2 + K$$

$u = x^2$
$du = 2x\,dx$ 1

1b) $$\int \frac{t^3}{1+t^4}\,dt = \frac{1}{4}\int \frac{4t^3}{1+t^4}\,dt \overset{1}{=} \frac{1}{4}\int \frac{du}{u} \overset{1}{=} \frac{1}{4}\ln\big(\underbrace{1+t^4}_{1+t^4>0}\big) + K$$

$1+t^4 = u$ 1
$du = 4t^3\,dt$

1c) $$\int \frac{e^{2x}}{\sqrt{1+e^x}}\,dx = \int \frac{2e^x e^x}{2\sqrt{1+e^x}}\,dx = \int 2(u^2-1)\,du = \frac{2}{3}u^3 - 2u + K$$
1
$$= \frac{2}{3}(1+e^x)^{3/2} - 2\sqrt{1+e^x} + K \quad 1$$

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$\sqrt{1+e^x} = u \Rightarrow$ 1 $\quad du = \frac{e^x}{2\sqrt{1+e^x}}$ $\quad \Big\| = \sqrt{1+e^x}\left(\frac{2}{3}(1+e^x) - 2\right) + K = \sqrt{1+e^x}\left(e^x - \frac{4}{3}\right) + K$

$e^x = u^2 - 1$

2a) $$\int 3z e^{2z}\,dz = \frac{3}{2}ze^{2z} - \int \frac{3}{2}e^{2z}\,dz = \frac{3}{2}ze^{2z} - \frac{3}{4}e^{2z} + K \overset{1}{=} \frac{3}{2}e^{2z}\left(z - \frac{1}{2}\right) + K$$

$f \quad g' \Rightarrow g = \frac{1}{2}e^{2z}$ $\quad$ 2

2b) $$\int e^x\cos x\,dx = e^x\cos x + \int e^x \sin x\,dx = e^x\cos x + \left(e^x\sin x - \int e^x\cos x\,dx\right)$$

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$g' \quad f$ (auch umgekehrt ok) $\qquad g' \quad f$ $\qquad$ 2

$$\Rightarrow 2\int e^x\cos x\,dx = e^x(\cos x + \sin x) + K \Rightarrow \int e^x\cos x\,dx \overset{1}{=} \frac{e^x}{2}(\sin x + \cos x) + K$$

3a) $$\frac{1}{y(y+1)} = \frac{A}{y} + \frac{B}{1+y} = \frac{A(1+y) + B(y)}{y(y+1)} \Rightarrow \begin{cases} A = 1 \\ A + B = 0 \Rightarrow B = -1 \end{cases} \quad 1$$

$$\int \frac{1}{y(1+y)}\,dy = \int \frac{1}{y}\,dy \overset{1}{-} \int \frac{1}{1+y}\,dy = \ln|y| - \ln|y+1| + K =$$
$$= \ln\left|\frac{y}{y+1}\right| + K \quad 1$$

3b) $$\int_1^e \frac{\sqrt{\ln x}}{x}\,dx \Rightarrow \quad \ln x = u \quad 1 \qquad \int \frac{\sqrt{\ln x}}{x}\,dx = \int \sqrt{u}\,du = \frac{2}{3}u^{3/2} + K \quad 1$$

$du = \frac{1}{x}\,dx$

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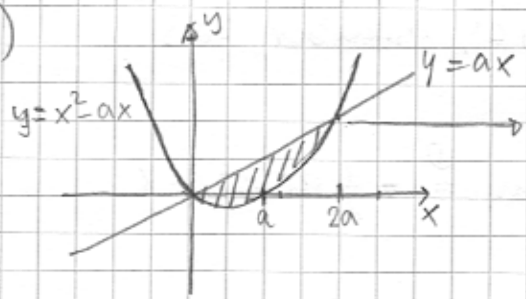
$$\int_1^e \frac{\sqrt{\ln x}}{x}\,dx = \frac{2}{3}(\ln x)^{3/2}\Big|_1^e = \frac{2}{3}\underbrace{(\ln e)^{3/2}}_{1} - \cancel{\frac{2}{3}\underbrace{(\ln 1)^{3/2}}_{0}} = \underline{\underline{\frac{2}{3}}} \quad 1$$

4)



$ax = x^2 - ax$

$x^2 - 2ax = 0$

$x(x-2a) = 0 \longrightarrow x = 0 \,,\; x = 2a$ (1)

$$\left|\int_0^{2a} ax - (x^2 - ax)\,dx\right| \underset{(1)}{=} \left|\int_0^{2a} (2ax - x^2)\,dx\right| \overset{(1)}{=} \left|\frac{2ax^2}{2} - \frac{x^3}{3}\right|_0^{2a} =$$

$$= \left|a(2a)^2 - \frac{(2a)^3}{3} - 0\right| \overset{(1)}{=} \left|4a^3 - \frac{8}{3}a^3\right| = \left|\frac{4}{3}a^3\right| = 36$$

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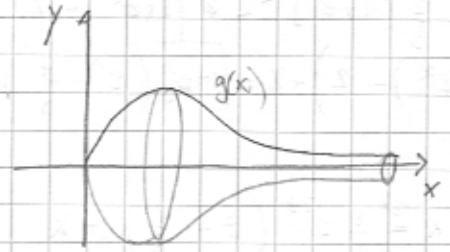
$$|a^3| \overset{(1)}{=} 36 \cdot \frac{3}{4} = 27 \implies a = \pm 3 \quad (1)$$

6)



NS: $g(x) = 0 \iff x = 0$

$$V = \pi \int_0^k g(x)^2\,dx = \pi \int_0^k 4 \cdot x \cdot e^{-x^2}\,dx = \quad (2)$$

$$= 2\pi \int_0^k 2x\, e^{-x^2}\,dx = 2\pi \int_0^{k^2} e^{-u}\,du = -2\pi\, e^{-u}\Big|_0^{k^2} = (*) \quad (1)$$

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$x^2 = u \quad du = 2x\,dx$

$x = 0 \implies u = 0 \qquad x = k \implies u = \sqrt{k}$

$$(*) = -2\pi\left(e^{-k^2} - e^0\right) = -2\pi\left(e^{-k^2} - 1\right) \quad (1)$$

$$\lim_{k\to\infty}\left(-2\pi\left(e^{-k^2} - 1\right)\right) \overset{(1)}{=} 2\pi \lim_{k\to\infty}\left(1 - \underbrace{\frac{1}{e^{k^2}}}_{\to 0}\right) \overset{(1)}{=} \underline{\underline{2\pi}}$$

7) a) $\operatorname{Max}(\sin k \cdot x) \overset{(1)}{=} 1 \implies$ Schnittpunkt in $(x_s, 1)$

$h(x) = 1 \implies x \overset{(1)}{=} 1 \quad$ " in $(1,1)$

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$$\sin(k \cdot 1) \overset{(1)}{=} 1 \implies k \cdot 1 = \frac{\pi}{2} \implies \underline{k = \frac{\pi}{2}} \quad (1)$$

b) $$A = \int_0^1 g(x)\,dx + \lim_{k\to\infty}\int_1^k h(x)\,dx \overset{(2)}{=} \int_0^1 \sin\left(\frac{\pi}{2} \cdot x\right)dx + \lim_{k\to\infty}\int_1^k \frac{1}{x^2}\,dx$$

$$\overset{(2)}{=} -\frac{2}{\pi}\cos\left(\frac{\pi}{2}x\right)\Big|_0^1 - \lim_{k\to\infty}\frac{1}{x}\Big|_1^k \overset{(1)}{=} -\frac{2}{\pi}(0 - 1) - \lim_{k\to\infty}\left(\frac{1}{k}^{\to 0} - 1\right) =$$

$$= \underline{\underline{\frac{2}{\pi} + 1}} \quad (1)$$

5) $\int \underset{f}{x} \cdot \underset{g'}{\sin^2 x}\, dx \quad \Rightarrow \quad \int \sin^2 x\, dx = \int \underset{f}{\sin x} \cdot \underset{g'}{\sin x}\, dx =$

$$= -\sin x \cos x + \int \cos x \cdot \cos x\, dx = -\sin x \cos x + \int (1 - \sin^2 x)\, dx$$

$$② \quad = -\sin x \cos x + x - \int \sin^2 x\, dx$$

$$\Rightarrow \int \sin^2 x\, dx = \frac{1}{2}\left(x - \sin x \cos x\right)$$

$$\int \underset{f}{x}\, \underset{g'}{\sin^2 x}\, dx \overset{①}{=} \frac{x}{2}\left(x - \sin x \cos x\right) - \int \left(\frac{x}{2} - \frac{\sin x \cos x}{2}\right) dx =$$

$$\underset{①}{=} \frac{x^2}{2} - \frac{x \sin x \cos x}{2} - \frac{x^2}{4} + \frac{1}{4}\int 2 \sin x \cos x\, dx =$$

$u = \cos x \rightarrow du = -\sin x\, dx$

$$= \frac{x^2}{4} - \frac{x}{2}\sin x \cos x + \frac{1}{4}\int 2u\, du \overset{①}{=} \underline{\underline{\frac{x^2}{4} - \frac{x}{2}\sin x \cos x - \frac{\cos^2 x}{4} + K}}$$

a) A_i sind im Intervall $[n\pi; (n+1)\pi]$ begrenzt:

$$\int_{n\pi}^{(n+1)\pi} x \sin^2 x\, dx = \underset{①}{\frac{x^2}{4} - \frac{x}{2}\sin x \cos x - \frac{\cos^2 x}{4}} \Bigg|_{n\pi}^{(n+1)\pi} = \frac{(n+1)^2\pi^2}{4} - \frac{(n\pi)^2}{4} \underset{①}{=}$$

- bei $k\pi$ ist $\sin x$ immer null
- $\cos^2 x$ ist immer 1 bei $k\pi$

$$= \cancel{\frac{n^2\pi^2}{4}} + \frac{\pi^2}{4} + \frac{2n\pi^2}{4} - \cancel{\frac{n^2\pi^2}{4}} \overset{①}{=} \frac{\pi^2}{4} + \frac{\pi^2}{2} \cdot n \Rightarrow a_n = a_1 + d \cdot n \quad \text{AF.} \quad (*)$$

b) $$\underset{①}{\int_{n\pi}^{(n+1)\pi} x\, dx} = \underset{①}{\frac{x^2}{2}\Bigg|_{n\pi}^{(n+1)^2\pi}} = \frac{(n+1)^2\pi^2}{2} - \frac{(n\pi)^2}{2} = \underset{①}{\frac{\pi^2}{2} + \pi^2 \cdot n} \Rightarrow \text{die Hälfte von } (*)$$

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TOT: 53P