TEIL A – 4C – 6.12.13 – MATHE

1) $h(x) = \left(1 - \frac{x}{k}\right)\sqrt{x}$

a) $V \overset{1}{=} \pi \int_0^k \left[\left(1 - \frac{x}{k}\right)\sqrt{x}\right]^2 dx \overset{1}{=} \pi \int_0^k \left(1 + \frac{x^2}{k^2} - 2\frac{x}{k}\right) x \, dx \overset{1}{=} \pi \int_0^k x + \frac{x^3}{k^2} - \frac{2x^2}{k} \, dx$

$\overset{1}{=} \pi \left[\frac{x^2}{2} + \frac{x^4}{4k^2} - \frac{2x^3}{3k}\right]_0^k = \pi k^2 \left[\frac{1}{2} + \frac{1}{4} - \frac{2}{3}\right] = \frac{1}{12}\pi k^2 \overset{!}{=} \frac{4}{3}\pi$ 1

6

$k^2 = \frac{12 \cdot 4}{3} = \frac{48}{3} = 16 \implies k = \pm 4$ aber $0 \leq k \implies \underline{\underline{k = 4}}$ 1

b) $k = 3 \implies h(x) = \left(1 - \frac{x}{3}\right)\sqrt{x}$

6 $\int_0^3 \left(1 - \frac{x}{3}\right)\sqrt{x} \, dx \overset{1}{=} \int_0^3 \left(\sqrt{x} - \frac{x\sqrt{x}}{3}\right) dx = \int_0^3 \left(x^{1/2} - \frac{x^{3/2}}{3}\right) dx =$

$\overset{1}{=} \frac{2}{3} x^{3/2} - \frac{2x^{5/2}}{5 \cdot 3} \Big|_0^3 = \frac{2}{3} 3^{3/2} - \frac{2}{15} 3^{5/2} = \frac{2}{3} \cdot 3\sqrt{3} - \frac{2}{15} \cdot 3^2\sqrt{3} \overset{1}{=}$

$\overset{1}{=} \sqrt{3}\left(2 - \frac{6}{5}\right) = \frac{4}{5}\sqrt{3}$ 1

2) $f''(x) = -\frac{2}{3}x \qquad f'(x) \overset{1}{=} -\frac{2}{3}\frac{1}{2}x^2 + K_1 \qquad f(x) = -\frac{1}{9}x^3 + K_1 x + K_2$ 1

a)

5 $f(x) = -f(-x) \implies K_2 = 0$ 1 $\qquad f(6) = -\frac{1}{9}6^3 + K_1 \cdot 6 = -6$

$K_1 = \frac{-6 + 6^3/9}{6} = -1 + 36/9 = 3$ 1

$f(x) = -\frac{1}{9}x^3 + 3x$ 1

b) $f(x) = 0 \quad x = 0$ oder $-\frac{1}{9}x^2 = -3 \quad x^2 = 27 \quad x = \pm\sqrt{27}$ 1

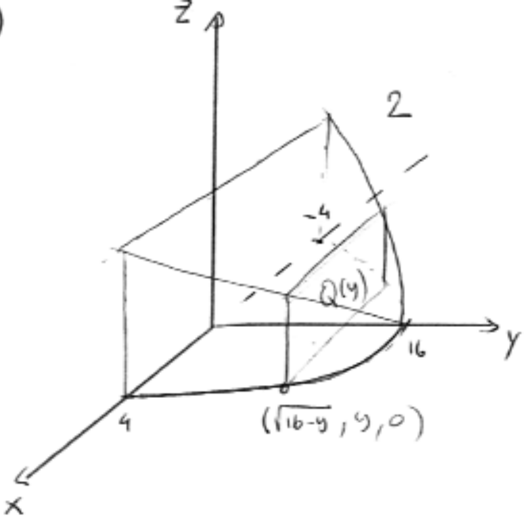
$F = 2 \cdot \int_0^{\sqrt{27}} -\frac{x^3}{9} + 3x \, dx = 2 \cdot \left(-\frac{x^4}{36} + \frac{3x^2}{2}\right)_0^{\sqrt{27}}$ 1 1 $=$

4

$= 2\left(-\frac{27^2}{36} + \frac{3 \cdot 27}{2}\right) = 2\left(-\frac{3^{6}}{3^2 \cdot 2^2} + \frac{3^4}{2}\right) = 2\left(\frac{81}{2} - \frac{81}{4}\right) = \frac{81}{2}$ 1

3)

6



$$Q(y) = 2\sqrt{16-y} \cdot 2\sqrt{16-y}$$

$$= 4(16-y) \quad 1$$

$$V = \int_0^{16} 4(16-y)\,dy \overset{1}{=} 4\left(16y - \frac{y^2}{2}\right)\Big|_0^{16} =$$

$$= 4\left(16^2 - \frac{16^2}{2}\right)_1 = 2^2\left(2^8 - 2^7\right) = 2^9 = \underset{1}{512}$$

A1) $p(X \leq 2{,}8\,kg) = 0{,}05$ (5^{o} Perzentil)

3 $\mu = 3{,}45\,kg$

3 b) $p(X \leq 2{,}8) = \phi\left(\frac{x-\mu}{\sigma}\right) = 0{,}05 \Rightarrow \sigma = \frac{x-\mu}{-x^*} = \frac{2{,}8 - 3{,}45}{-1{,}645}\,kg = 395\,g$

3 c) $\phi\left(\frac{x-\mu}{\sigma}\right) = \phi\left(\frac{7{,}5-9{,}2}{1{,}05}\right) = p(x \leq 7{,}5\,kg) = 1 - \phi\left(\frac{9{,}2-7{,}5}{1{,}05}\right) = 1 - \phi(1{,}62) = 5{,}3\%$

1 d) Keine Sorgen! :) Sie wächst der Kurve entlang!

B1) $p(X > 220) = 1 - p(X \leq 220) = 1 - \phi\left(\frac{220-205}{52}\right) = 1 - \phi(0{,}288) = 1 - 0{,}6141 =$
3 $= 38{,}6\%$

5 b) $p(170 \leq X \leq 220) = \phi\left(\frac{220-205}{52}\right) - \phi\left(\frac{169-205}{52}\right) =$

$= \phi\left(\frac{220-205}{52}\right) - 1 + \phi\left(\frac{205-169}{52}\right) = \phi(0{,}67) + \phi(0{,}69) - 1$

$= 0{,}7486 + 0{,}7549 - 1 = 50{,}4\%$

c) $E = \mu = np \rightarrow p = \mu/n = \frac{205}{300000} = 6{,}83 \cdot 10^{-4}$

3 $\sigma = \sqrt{np(1-p)} = \sqrt{300000 \cdot \frac{205}{300000}\left(1 - \frac{205}{300000}\right)} = 14{,}3$

d) $p(X > 220) = 1 - \text{binomcdf}\left(300000, \frac{205}{300000}, 220\right) = 13{,}8\%$

3
- Kann man wirklich jeder Person eine WK zuordnen, dass sie dorthin geht?
- Das mögliche "Publikum" ist deutlich geringer als 300'000!
- Der Wirt weiss aus Erfahrung was μ und σ sind. (Sollten stimmen!)
- Entscheiden die Leute wirklich unabhängig? Wetter? Gruppendynamik?