TAYLORENTWICKLUNG — MD — 4B

1)a) $p(x) = 4 + 6(x-1) + \frac{8}{2}(x-1)^2 + \frac{6}{3!}(x-1)^3 = 4 + 6(x-1) + 4(x-1)^2 + (x-1)^3$

$f'(x) = 3x^2 + 2x + 1 \qquad f'(1) = 6 \qquad f(1) = 4$

$f''(x) = 6x + 2 \qquad f''(1) = 8$

$f'''(x) = 6 \qquad f'''(1) = 6$

b) $f'(x) = \frac{-\sin(\sin x)\cdot \cos x}{\cos(\sin x)} = -\cos x \cdot \tan(\sin x) \qquad f'(0) = 0 \qquad f(0) = 0$

$f''(x) = \sin(x)\cdot \tan(\sin x) - \cos^2(x) \cdot \frac{1}{\cos^2(\sin x)} \qquad f''(0) = -1$

$f'''(x) = \cos x \tan(\sin x) + \frac{3\sin x \cos x}{\cos^2(\sin x)} - \frac{2\cos^3(x)\tan(\sin x)}{\cos^2(\sin x)}, \qquad f'''(0) = 0$

$\Rightarrow \quad p(x) = -\frac{x^2}{2}$

2) $f(1) = \frac{1}{2} \qquad f'(x) = \frac{-1}{(1+x)^2} \qquad f''(x) = \frac{2}{(1+x)^3} \qquad f'''(x) = \frac{-6}{(1+x)^4}$

$f'(1) = -\frac{1}{4} \qquad f''(1) = \frac{1}{4} \qquad f'''(1) = \frac{-3}{8}$

$p(x) = \frac{1}{2} - \frac{1}{4}(x-1) + \frac{1}{4}\cdot\frac{1}{2}(x-1)^2 - \frac{3}{8}\cdot\frac{1}{3!}(x-1)^3 + \dots$

$= \frac{1}{2} - \frac{1}{4}(x-1) + \frac{1}{8}(x-1)^2 - \frac{1}{16}(x-1)^3 + \dots = \sum_{k=0}^{\infty} \frac{-(x-1)^k}{(-2)^{k+1}}$

3) $f(0) = 0 \qquad f'(x) = -e^{-x}(x-1) \qquad f''(x) = e^{-x}(x-2) \qquad f'''(x) = -e^{-x}(x-3)$

$f'(0) = +1 \qquad f''(0) = -2 \qquad f'''(0) = +3$

$p(x) = 1\cdot x - \frac{2}{2}\cdot x^2 + \frac{3}{3!}x^3 = x - x^2 + \frac{x^3}{2}$

4)a) $\lim_{x\to 0} \frac{\sin ax}{\sin bx} = \lim_{x\to 0} \frac{ax - \frac{(ax)^3}{3!}}{bx - \frac{(bx)^3}{3!}} = \frac{a}{b}$

b) $\lim_{x\to 0} \frac{e^{x} - e^{-x}}{x} = \lim_{x\to 0} \frac{(\cancel{1} + x + \cancel{\frac{x^2}{2}}) - (\cancel{1} - x + \cancel{\frac{(-x)^2}{2}})}{x} = 2$

c) $\lim_{x\to 0} \frac{x - \sin x}{x^3} = \lim_{x\to 0} \frac{\cancel{x} - (\cancel{x} - \frac{x^3}{3!} + \frac{x^5}{5!})}{x^3} = \frac{1}{6}$

5) e^2 als Zahlenreihe $\rightarrow$ Taylorentwicklung in 0

$$p(x) = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \frac{x^4}{4!} + \frac{x^5}{5!} + \dots$$

$$= 1 + x + \frac{x^2}{2} + \frac{x^3}{6} + \frac{x^4}{24} + \frac{x^5}{120} + \dots = \sum_{k=0}^{\infty} \frac{x^k}{k!}$$

$$p(2) = 1 + 2 + \frac{4}{2} + \frac{8}{6} + \frac{16}{24} + \frac{32}{120} + \dots = 1 + 2 + 2 + \frac{4}{3} + \frac{2}{3} + \frac{4}{15} = \sum_{k=0}^{\infty} \frac{2^k}{k!}$$

6) $f(0) = \sqrt{1} = 1$

$$f'(0) = \frac{-1}{2\sqrt{1-x}}\Big|_{x=0} = -\frac{1}{2}$$

$$f''(0) = \frac{-1}{4(1-x)^{3/2}}\Big|_{x=0} = -\frac{1}{4}$$

$$f'''(0) = \frac{-3}{8(1-x)^{5/2}}\Big|_{x=0} = -\frac{3}{8}$$

$$f^{IV}(0) = \frac{-15}{16(1-x)^{7/2}}\Big|_{x=0} = -\frac{15}{16} \qquad f^{V}(0) = \frac{-105}{32(1-x)^{9/2}}\Big|_{x=0} = -\frac{105}{32}$$

$$p(x) = 1 - \frac{1}{2}\frac{x}{1} - \frac{1}{4}\frac{x^2}{2!} - \frac{3}{8}\frac{x^3}{3!} - \frac{15}{16}\cdot\frac{x^4}{4!} - \frac{105}{32}\cdot\frac{x^5}{5!} + \dots$$

$$p(0{,}05) = 1 - \frac{1}{2}\,0{,}05 - \frac{1}{4}\cdot\frac{0{,}05^2}{2} - \frac{3}{8}\cdot\frac{0{,}05^3}{6} - \frac{15}{16}\cdot\frac{0{,}05^4}{24} - \frac{105}{32}\cdot\frac{0{,}05^5}{120}$$

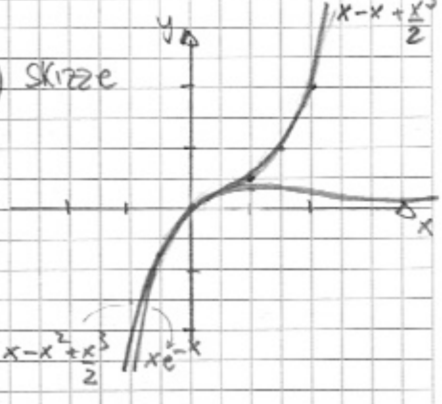
$$= 1 - \frac{1}{40} - \frac{1}{3200} - \frac{1}{128000} - \frac{1}{4096000} - \frac{1792}{3200000}$$

$$= \underbrace{1 - 0{,}025 - 0{,}0003125 - 0{,}0000078125}\dots$$

(erste 4 Terme sind auf mindestens 8 Ziffer genau)

$$= 0{,}9746796875$$

3) Skizze



7) $f(2) = \frac{1}{-1} = -1$

$$f'(2) = \frac{1}{(1-x)^2}\Big|_{x=2} = 1$$

$$f''(2) = \frac{-2}{(x-1)^3}\Big|_{x=2} = -2$$

$$f'''(2) = \frac{6}{(1-x)^4}\Big|_{x=2} = \frac{6}{(-1)^4} = 6$$

$$p(x) = -1 + 1(x-2) - 2\frac{(x-2)^2}{2} + \frac{6(x-2)^3}{3!}$$

$$= -1 + (x-2) - (x-2)^2 + (x-2)^3 = \sum_{k=0}^{\infty} (-1)^{k+1}(x-2)^k$$