WK – Verteilungen 2 4B MD

1. $P(x) = \frac{\mu^x}{x!} \cdot e^{-\mu}$ $\mu = 1$

$p(0) = \frac{1^0}{0!} e^{-1} = e^{-1} = 36{,}8\% = P(1) = \frac{1^1}{1!} e^{-1}$

$p(2) = \frac{1^2}{2!} e^{-2} = \frac{e^{-1}}{2} = 18{,}4\%$ $P(3) = \frac{1^3}{3!} \cdot e^{-1} = \frac{e^{-1}}{6} = 6{,}1\%$

$n = p(3) \cdot N = \frac{1}{6e} \cdot 365 = 22$

2) $p(x > 2) = 1 - p(0) - p(1) - p(2) = 1 - \frac{3^0}{0!} e^{-3} - \frac{3^1}{1!} e^{-3} - \frac{3^2}{2!} e^{-3} =$

$= 1 - \frac{1}{e^3}\left(1 + 2 + \frac{9}{2}\right) = 57{,}7\%$

3) $p = 5\%$ $\sigma = \sqrt{np(1-p)} = \sqrt{10 \cdot 0{,}05 \cdot 0{,}95} = 0{,}7 < 3 \Rightarrow$

$n = 10$ Normalvert. nicht geeignet

$p = 5\%$ nicht besonders klein → Poisson nicht geeignet.

→ Binomialverteilung

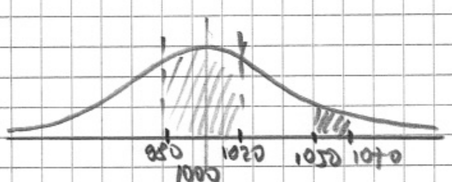
$p(x \geq 2) = 1 - p(0) - p(1) = 1 - \binom{10}{0} \cdot 0{,}05^0 \cdot 0{,}95^{10} - \binom{10}{1} \cdot 0{,}05^1 \cdot 0{,}95^9 =$

$= 1 - 0{,}95^{10} - 10 \cdot 0{,}05 \cdot 0{,}95 = 8{,}6\%$

Mit Poisson: $p(x \geq 2) = 1 - \frac{0{,}5^0}{0!} e^{-0{,}5} - \frac{0{,}5^1}{1!} e^{-0{,}5} = 9{,}0\%$

Mit Normalv: $p(x \geq 2) = 1 - p(x \leq 1) = 1 - \phi\left(\frac{1{,}5 - 0{,}5}{\sqrt{10 \cdot 0{,}05 \cdot 0{,}95}}\right) =$

$= 1 - \phi(1{,}451) = 7{,}4\%$

4) 

$\mu = 1000$ $\sigma = 30$

a) $p(1050 \leq x \leq 1070) = \phi\left(\frac{1070{,}50 - 1000}{30}\right) - \phi\left(\frac{1049{,}5 - 1000}{30}\right) =$

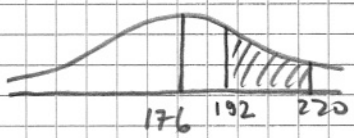
$= \phi(2{,}35) - \phi(1{,}65) = 0{,}99061 - 0{,}9505 = 4{,}0\%$

b) $p\left(980 \leq X \leq 1020\right) = \phi\left(\frac{1020{,}5 - 1000}{30}\right) - \phi\left(\frac{979{,}5 - 1000}{30}\right) =$

$= \phi\left(\frac{41}{60}\right) - \left(1 - \phi\left(\frac{41}{60}\right)\right) = 2\phi\left(\frac{41}{60}\right) - 1 = 2\phi\left(0{,}683\right) - 1 =$

$= 2 \cdot 0{,}7517 - 1 = 50{,}3\,\%$

5)



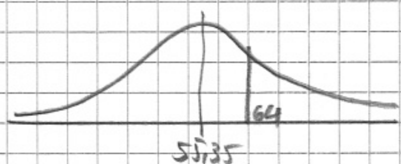
$n = 220$

$p = 0{,}80$ $\Rightarrow$ $\mu = n \cdot p = 176$

$p\left(192 \leq X \leq 220\right) = \phi\left(\frac{220{,}5 - 220 \cdot 0{,}8}{\sqrt{220 \cdot 0{,}8 \cdot 0{,}2}}\right) - \phi\left(\frac{191{,}5 - 220 \cdot 0{,}8}{\sqrt{220 \cdot 0{,}8 \cdot 0{,}2}}\right) =$

$= \phi\left(6{,}83\right) - \phi\left(2{,}61\right) = 1 - 0{,}99547 = 4{,}5\,‰$

6)



$p = 0{,}45$

$n = 123$

$x = 64$

$P\left(X \geq 64\right) = 1 - \phi\left(\frac{63{,}5 - 123 \cdot 0{,}45}{\sqrt{123 \cdot 0{,}45 \cdot 0{,}55}}\right) = 1 - \phi\left(1{,}477\right) = 6{,}9\,\%$

7) $p = 8\%$ $\quad 1 - p = 0{,}92$

$p\left(X \leq 400\right) = \phi\left(\frac{400{,}5 - 420 \cdot 0{,}92}{\sqrt{420 \cdot 0{,}92 \cdot 0{,}08}}\right) = \phi\left(2{,}54\right) = 99{,}4\,\%$

8) $\begin{cases} p\left(X > 120\right) = 0{,}12 = 1 - \phi\left(\frac{120-\mu}{\sigma}\right) & \Rightarrow \quad \phi\left(\frac{120-\mu}{\sigma}\right) = 0{,}88 \\ p\left(X \leq 90\right) = 0{,}20 = \phi\left(\frac{90-\mu}{\sigma}\right) & \Rightarrow \quad \phi\left(\frac{\mu-90}{\sigma}\right) = 0{,}8 \end{cases}$

$\begin{cases} \frac{120-\mu}{\sigma} = 1{,}175 \\ \frac{\mu-90}{\sigma} = 0{,}84 \end{cases} \quad \begin{cases} 120 - \mu = 1{,}175\sigma \\ -90 + \mu = 0{,}84\sigma \end{cases} \rightarrow \quad \begin{array}{l} 30 = 2{,}015\sigma \\ \sigma = 14{,}9 \end{array}$

$\mu = 0{,}84 \cdot \sigma + 90 = 103$

9) $p\left(X > 13\right) = 1 - \phi\left(\frac{13{,}5 - 200 \cdot 0{,}04}{\sqrt{200 \cdot 0{,}04 \cdot 0{,}96}}\right) = 1 - \phi\left(1{,}985\right) = 2{,}4\,\%$