WECHSELSTROM

1) $Z = \sqrt{R^2 + (\omega L)^2}$ und $\tan\varphi = \frac{\omega L}{R} \Rightarrow L = \frac{R \tan\varphi}{\omega}$

$= \sqrt{R^2 + R^2 \tan^2\varphi}$

$\hat{I} = I\sqrt{2} = \sqrt{2}\,\frac{U}{Z} = \sqrt{2}\,\frac{U}{R\sqrt{1+\tan^2\varphi}} = \frac{\sqrt{2}}{30\Omega} \cdot \frac{230V}{\sqrt{1+\tan^2 55}} = 6{,}2\,A$

2a) $L = \mu_0 \frac{N^2 A}{l} = \frac{4\pi \cdot 10^{-7}\,\frac{Vs}{Am} \cdot 1000^2 \cdot \pi\,(0{,}015m)^2}{1{,}0m} = 0{,}89\,mH$

b) $I = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + (\omega L)^2}} = \frac{U}{\sqrt{R^2 + \frac{4\pi^2 f^2 \mu_0^2 N^4 \pi^2 r^4}{l^2}}} =$

$= \frac{2{,}8V}{\sqrt{(2\Omega)^2 + \left(\frac{4\pi^2 (350Hz \cdot 1000^2 \cdot 4\pi \cdot 10^{-7}\frac{Vs}{Am} \cdot \pi^2 \cdot (0{,}015m)^2}{(1{,}0m)^2}\right)^2}} = 1{,}0A$

c) $\varphi = \arctan\frac{\omega L}{R} = \arctan\frac{2\pi f \cdot \mu_0 N^2 \pi (r)^2}{lR} =$

$= \arctan\left(\frac{2\pi \cdot 350Hz \cdot 4\pi \cdot 10^{-7}\frac{Vs}{Am} \cdot 1000^2 \cdot \pi \cdot (0{,}015m)^2}{1{,}0m \cdot 2\Omega}\right) = 44{,}3^\circ$

d) $P_1 = \frac{U_R^2}{R} = I_R^2 \cdot R = \frac{U^2 R}{R^2 + \frac{4\pi^2 f^2 \mu_0^2 N^4 \pi^2 r^4}{l^2}} = \ldots = 2100\,W$

$P_2 = U \cdot I \cdot \cos\varphi = \frac{U^2 \cdot \cos\arctan\left(\frac{2\pi R \mu_0 N^2 \pi r^2}{lR}\right)}{\sqrt{R^2 + \frac{4\pi^2 f^2 \mu_0^2 N^4 \pi^2 r^4}{l^2}}} = \ldots\ 2{,}00\,W$

3) $f_1 = 2kHz$ $\quad R = 10\Omega$

$I_1 = 78mA$ $\quad U = 8V$

$\varphi_2 = 45^\circ$

$I = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + (\omega L)^2}} \Rightarrow L = \frac{1}{2\pi f_1}\sqrt{\left(\frac{U}{I_1}\right)^2 - R^2}$

$\tan\varphi_2 = \frac{\omega L}{R} \qquad L = \frac{R \tan\varphi}{2\pi f_2}$

$$\frac{R \tan\varphi_2}{2\pi f_2} = \frac{1}{2\pi f_1}\sqrt{\frac{U^2}{I^2} - R^2}$$

$$f_2 = \frac{f_1 R \tan\varphi_2}{\sqrt{\frac{U^2}{I^2} - R^2}} = \frac{2\,\text{kHz} \cdot 10\,\Omega \cdot 1}{\sqrt{\left(\frac{8\,\text{V}}{0{,}03\,\text{A}}\right)^2 - (10\,\Omega)^2}} = 196\,\text{Hz}$$

4) a) $Z = \sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \sqrt{(5{,}8\,\Omega)^2 + \left(2\pi \cdot 4500\,\frac{1}{s} \cdot 0{,}0031\,\text{H} - \left(2\pi \cdot 4500\,\frac{1}{s} \cdot 0{,}37\,\mu\text{F}\right)^{-1}\right)^2}$

$= 9{,}8\,\Omega$

b) $I_{\text{eff}} = \frac{U}{Z} = \frac{U}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \ldots = 0{,}75\,\text{A}$

$\hat{I} = \frac{U}{Z}\sqrt{2} = \frac{U\sqrt{2}}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \ldots = 1{,}1\,\text{A}$

c) $\varphi = \arctan\frac{\omega L - \frac{1}{\omega C}}{R} = \arctan\frac{2\pi \cdot 4500\,\frac{1}{s} \cdot 31\,\text{mH} - \left(2\pi \cdot 4500\,\frac{1}{s} \cdot 0{,}37\,\mu\text{F}\right)^{-1}}{5{,}8\,\Omega}$

$= -53{,}8° = -0{,}94\,\text{rad}$

d) $S = U \cdot I = \frac{U^2}{Z} = \frac{U^2}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} = \ldots = 5{,}6\,\text{VA}$

$Q = U \cdot I \cdot \sin\varphi = \frac{U^2}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \sin\left(\arctan\frac{\omega L - \frac{1}{\omega C}}{R}\right) = \ldots = 4{,}5\,\text{VAR}$

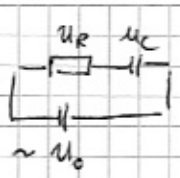
$P = U \cdot I \cdot \cos\varphi = \frac{U^2}{\sqrt{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2}} \cos\left(\arctan\frac{\omega L - \frac{1}{\omega C}}{R}\right) = \ldots = 3{,}3\,\text{W}$

e) Wirkleistung $P = U_R \cdot I_R = R \cdot I^2 = R \cdot \frac{U^2}{Z^2} = \frac{R \cdot U^2}{R^2 + \left(\omega L - \frac{1}{\omega C}\right)^2} = \ldots 3{,}3\,\text{W}$

f) $I_{\max} \Leftrightarrow Z_{\min} \Leftrightarrow \omega L = \frac{1}{\omega C}$

$\Rightarrow 4\pi^2 f^2 = \frac{1}{LC}$ $\quad f = \sqrt{\frac{1}{4\pi^2 LC}} = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{31\,\text{mH} \cdot 0{,}37\,\mu\text{F}}} = 4{,}7\,\text{kHz}$

g) $I = \frac{U}{Z_{\min}} = \frac{U}{R} = \frac{7{,}4\,\text{V}}{5{,}8\,\Omega} = 1{,}3\,\text{A}$

6) a) 

$u_C = \sqrt{u_0^2 - u_R^2}$ (Pythagoras) $\quad u_C = Z_C \cdot I$

$P_R = u_R I_R = u_R \cdot I = \frac{u_R^2}{R} \quad = \frac{I}{\omega C}$

$$C = \frac{I}{\omega u_C} = \frac{I}{2\pi f \sqrt{u_0^2 - u_R^2}} = \frac{P_R}{2\pi f u_R \sqrt{u_0^2 - u_R^2}} =$$

$$= \frac{40\,W}{2\pi \cdot 50 \frac{1}{s} \cdot 110\,V \sqrt{(230\,V)^2 - (110\,V)^2}} = 5{,}7\,\mu F$$

$$Q = U \cdot I \cdot \sin\varphi = U \cdot I \cdot \sin\left(\arctan \frac{1/\omega C}{R}\right) =$$

$$= \frac{u_0^2}{\sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}} \cdot \sin\left(\arctan \frac{u_C}{R}\right) = \frac{u_0^2}{\sqrt{R^2 + \frac{1}{(\omega C)^2}}} \sin \arctan\left(\frac{\sqrt{u_0^2 - u_R^2}}{u_R}\right) =$$

$$= \frac{(u_0)^2}{\sqrt{\left(\frac{u_R^2}{P_R}\right)^2 + \left(\frac{u_R \sqrt{u_0^2 - u_R^2}}{(P_R)}\right)^2}} \cdot \sin\left(\arctan \frac{\sqrt{u_0^2 - u_R^2}}{u_R}\right) = \ldots = 73\,W$$

5) b) $P = u_R \cdot I = 104{,}5\,V \; 0{,}455\,A = 47{,}5\,W$

c) sollte nach Pythagoras addiert werden.

$$\sqrt{u_R^2 + u_L^2} = \sqrt{(104{,}5\,V)^2 + (174{,}3\,V)^2} = 203{,}2\,V < 224{,}3\,V$$

$\Rightarrow$ Innenwiderstand der Drossel nicht vernachlässigbar klein

d) $\varphi = \arctan \frac{u_L}{u_R} = \arctan \frac{174{,}3\,V}{104{,}5\,V} = \underline{1{,}031\,rad}$

$$\cong \arctan \frac{\sqrt{u_0^2 - u_R^2}}{u_R} = \frac{\sqrt{(224{,}3\,V)^2 - (104{,}5\,V)^2}}{104{,}5\,V} = \underline{1{,}086\,rad}$$

$$\cong \arctan \frac{u_L}{\sqrt{u_0^2 - u_L^2}} = \arctan \frac{(174{,}3\,V)}{\sqrt{(224{,}3\,V)^2 - (174{,}3\,V)^2}} = \underline{0{,}890\,rad}$$

e) $\frac{\omega L}{R} = \frac{u_L}{u_R} \qquad L = \frac{R u_L}{2\pi f u_R} = \frac{u_R u_L}{2\pi f I u_R} = \frac{u_L}{2\pi f I} \cong \frac{174{,}3\,V}{2\pi \cdot 50\,Hz \cdot 0{,}455\,A} = \underline{1{,}22\,H}$

$R = \frac{u_R}{I}$

$$L \cong \frac{\sqrt{u_0^2 - u_R^2}}{2\pi f I} = \frac{\sqrt{(224{,}3\,V)^2 - (104{,}5\,V)^2}}{2\pi \cdot 50\,Hz \cdot 0{,}455\,A} = \underline{1{,}39\,H}$$